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| **Mathematics** | | |
| **Sr. No.** | **Core Areas** | **Percentage** |
| 1 | Foundation of Mathematics | 4% |
| 2. | Elementary Mathematics | 8% |
| 3. | Calculus | 14% |
| 4. | Algebra | 8% |
| 5. | Complex Analysis | 7% |
| 6. | Differential Equation | 12% |
| 7. | Analytic Geometry | 4% |
| 8. | Vectors | 7% |
| 9. | Number Theory | 4% |
| 10. | Real Analysis | 6% |
| 11. | Functional Analysis | 6% |
| 12. | Computational Mathematics | 9% |
| 13. | Mechanics | 6% |
| 14 | Mathematical Statistics & Probability | 5% |
|  | **Total** | **100%** |

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| **Mathematics (Detailed)** | | |
| **Sr. No.** | **Core Areas** | **Percentage** |
| **1.** | **Foundation of mathematics:**  Basic notions, set operations, extended-set operations, indexed family of sets, countable and uncountable sets, relations, cardinality, equivalence relations, congruence, partitions, partial order, representation of relations, mathematical induction. Axiom of choice, zorn’s lamma, continuum hypothesis, propositional calculus, predicate calculus, law of excluded middle | **4%** |
| **2.** | **ELEMENTARY MATHEMATICS:**  **2.1 Elementary Mathematics-I \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_4%**  Sets, types of sets, Basic operations on sets, Venn diagrams, verification of commutative, associative, distributive and De Morgan’s laws through sets and Venn diagrams, whole numbers, integers, factors and multipliers, fractions, decimals, percentages, unitary method, BODMAS rule, ratio and proportion, financial arithmetic, algebraic operations, linear equations, distance, time and temperature, line segments, types of angles, types of triangles, types of quadrilaterals, perimeter and area, average, graphs (block, column, bar and pie)  **2.2 Elementary Mathematics-II \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_4%**  Rational numbers, real numbers, number systems with bases 2 and 10 and their conversions, exponents, square root of positive numbers, cubes and cube roots, HCF and LCM (using division and prime factorization) direct and inverse relations, taxes, profit, loss, discount and markup, compound proportion, income tax, Zakat and Ushr, operations with polynomials, algebraic identities involving , , and , factorization of algebraic expressions, simultaneous equations, solution by comparison, substitution, elimination and cross-multiplication, properties of angles, congruent and similar figures, congruent triangles, circumference and area of a circle, surface area and volume of sphere and cylinder, frequency distribution | **8%** |
| **3.** | **CALCULUS:**   * 1. **Differential Calculus\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_6%**   Functions, Inverse functions, Range and domain, Properties of functions, Composite functions, Even and odd functions, Shift of graphs, Types of functions, parametric equations, Limits, Infinite limit, vertical and horizontal asymptotes, Continuity, Piece wise continuity, and uniform continuity, infinite discontinuity, limit at infinity  Differentiability, Differentiation, Techniques of differentiation, Applications of differentiation (monotonicity of functions, critical points and points of inflections, extrema both relative and absolute, Optimization problems) Mean value theorem, Taylor’s theorem, Taylor series, Curve sketching, Implicit differentiation   * 1. **Integral Calculus 4%**   Integral, definite and indefinite integrals, proper and improper integrals, Fundamental theorems of calculus, Techniques of integration, Applications of definite integrals (area, arc length)   * 1. **Vector Calculus: 4%**   Vectors and analytic geometry of 2 and 3 dimensional spaces, vector-valued functions and space curves, functions of several variables, limits and continuity, partial derivatives, the chain rule, double and triple integrals with applications, line integrals, the Green’s theorem, surface area and surface integrals, the Green, the divergence and the Stokes theorems | **14%** |
| **4.** | **ALGEBRA:**  **4.1 Group Theory\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_2%**  Basic axioms of a group, abelian groups, center of a group, subgroups, cyclic groups, cosets and quotient sets, Lagrange’s theorem, even and odd permutations, cycles, length of a cycle, transpositions, symmetric and non-symmetric groups, alternating groups, normalizers and centralizers of a subset of a group, congruency classes of a group, normal subgroup, quotient groups, conjugacy, homomorphism and isomorphism between groups, homomorphism and isomorphism theorems, group of automorphisms, finite p-groups, internal and external direct products, group action on sets, orbits, 1st, 2nd and 3rdSylow theorems  **4.2 Algebra of Matrices\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_2%**  Matrix and its types, determinants and its properties, inverse of a matrix, row and column operations, echelon and reduced echelon form, rank of a matrix, consistent and inconsistent systems (conditions for the existence of zero, one or infinite solutions), solution of non-homogenous equations (Gauss-elimination method, Gauss-Jordon method, inverse method, Cramer’s rule), solution of homogenous equations and eigenvalue problems, groups and subgroups of matrices  **4.3 Ring Theory\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_2%**  Rings and its types (matrix rings, rings of endomorphisms, polynomial rings), integral domain, fields, characteristic of a ring, ideals, types of ideals, quotient rings, homomorphism of rings, isomorphism theorem of rings  **4.4 Linear Algebra\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_2%**  Vector spaces, subspaces, linear span of subset of a vector space, bases and dimension of a vector space, sums and direct sums of subspaces of a finite dimensional vector space, dimension theorem, linear transformation, null space, image space of a linear transformation, matrix of a linear transformation, rank and nullity of a linear transformation, relation between rank, nullity and dimension of domain of a linear transformation, orthogonal transformation, change of basis, inner-product spaces, projection of a vector on another vector, norm of a vector, Cauchy-Schwartz inequality, orthogonal and orthonormal bases, similar matrices and diagonalization of a matrix, Hom(V, W), dimension and basis of Hom(V, W), dual space and dual basis, annihilators | **8%** |
| **5.** | **COMPLEX ANALYSIS:**  **5.1 Complex Numbers and complex functions\_\_\_\_\_\_\_\_\_\_\_\_2%**  The algebra and the geometry of complex numbers. Complex valued functions and their types, elementary functions, complex exponents, limit continuity and differentiability  **5.2 Complex Analysis\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_2%**  Analytic functions, Cauchy-Riemann equations, harmonic functions, contours and contour integrals, the Cauchy-Goursat Theorem, the Cauchy integral formulae, the Morera Theorem, maximum modulus principle, the Liouville theorem, fundamental theorem of algebra  **5.3 Series & Integrals\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_3%**  Convergence of sequences and series, the Taylor series, power series representation of analytic functions, the Laurent series, branch point, zeros of analytic functions, residues and poles, the residue theorem, evaluation of improper integrals involving trigonometric functions | **7%** |
| **6.** | **DIFFERENTIAL EQUATIONS:**   * 1. **Ordinary Differential Equations\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_6%**   Formation and solution of first-order-differential equations, formation and solution of second and higher-order-linear-differential equations; differential equations with variable coefficients, Sturm-Liouville (S-L) system ,initial and boundary-value problems, series solution, the Frobenius method, solution of the Bessel, the hypergeometric, the Legendre and the Hermite equations, properties of the Bessel functions   * 1. **Partial Differential Equations\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_6%**   First-order-partial-differential equations, classification of second-order partial-differential equations, canonical form for second-order equations; wave, heat and the Laplace equations in Cartesian, cylindrical and spherical-polar coordinates; solution of partial-differential equation by the methods of: separation of variables, the Fourier, the Laplace and the Hankel transforms, non-homogeneous-partial-differential equations | **12%** |
| **7.** | **Analytic GEOMETRY:**  **7.1 Analytical Geometry: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_2%**  Cartesian-coordinate system, slope of a straight line, equation of a straight line, parallel and perpendicular lines, various forms of equation of a line, intersection of two lines, angle between two lines, distance between two points, distance between a point and a line, equation of a circle, circles formed under various conditions, intersection of lines and circles  **7.2 Conic Sections\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_2%**  Conic section, the general-second-degree equation. (circle, parabola, ellipse and hyperbola) and their properties | **4%** |
| **8.** | **VECTORS:**  **8.1 Vector Analysis\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_4%**  Vectors scalar, products, scalar- and vector-triple products, scalar- and vector-point functions, differentiation and integration of vectors, line integrals, path independence, surface integrals, volume integrals, gradient, divergence and curl with physical significance, vector identities, Green’s theorem in a plane, divergence theorem, Stokes’ theorem, coordinate systems and their bases, the spherical-polar- and the cylindrical-coordinate systems  **8.2 Tensor Analysis\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_3%**  Tensors of zero, first, second and higher orders, algebra of tensors, contraction of tensor, quotient theorem, symmetric and skew-symmetric tensors, invariance property, tensors in modeling anisotropic systems, physical tensors (moment of inertia, index of refraction), diagnolization of inertia tensor as aligning coordinate frame with natural symmetries of the system | **7%** |
| **9.** | **NUMBER THEORY:**  Divisibility, euclidean algorithm, GCD and LCM of 2 integers, properties of prime numbers, fundamental theorem of arithmetic, congruence relation, residue system, Euler’s phi-function, solution of system of linear congruence’s, Chinese remainder theorem, Fermat little theorem, Wilson theorem, primitive roots and indices; integers belonging to a given exponent (mod p), primitive roots of prime and composite moduli, indices, solutions of congruence’s using indices | **4%** |
| **10.** | **REAL ANALYSIS:**  **10.1 Theoretical Basis\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_3%**  Ordered sets, supremum and infimum, accumulation point, completeness properties of the real numbers, limits and continuity, properties of continuous functions on closed bounded intervals, derivatives, the mean value theorem, sequences and series of functions, power series, point-wise and uniform convergence, functions of several variables, open and closed sets and convergence of sequences in Rn; compactness, limits and continuity in several variables    **10.2 Integration Theory\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_3%**  Series of numbers and their convergence, Darboux upper and lower sums and integrals, Dabroux integrability, Riemann sums and Riemann integrals, Riemann integration, change of order of variables of integration, Riemann-Steiltjes integration, functions of bounded variation | **6%** |
| **11.** | **FUNCTIONAL ANALYSIS:**  **11.1 Metric Spaces\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_2%**  Concept and properties of metric space, Cauchy sequence, completeness and convergence  **11.2 Normed Spaces\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_2%**  Linear spaces, normed spaces, Banach spaces, Bounded and continuous linear operators and functionals, dual spaces, finite-dimensional spaces, F. Riesz lemma. The Hahn Banach theorem for complex spaces, the HB theorem for normed spaces, the open mapping theorem, the Banach-fixed-point theorem  **11.3 Inner-Product Spaces\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_2%**  Inner-product space, Hilbert space, orthogonal and orthonormal sets, orthogonal complements, Gram-Schmidt orthogonalization process, Reiz-representation theorem | **6%** |
| **12.** | **COMPUTATIONAL MATHEMATICS:**  **12.1 Solution of Non linear Equations\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_3%**  Computer arithmetic, approximations and errors; methods for the solution of nonlinear equations and their convergence: bisection method, regulafalsi method, fixed point iteration method, Newton-Raphson method, secant method; error analysis for iterative methods. Interpolation and polynomial approximation: Lagrange interpolation  **12.2 Solution of systems of linear equations\_\_\_\_\_\_\_\_\_\_\_\_\_3%**  Numerical solution of systems of algebraic linear equations, Gauss-elimination method, Gauss-Jordan method; matrix inversion; LU-factorization; Doolittle’s, Cholesky’s methods; Gauss-Seidel and Jacobi methods  **12.3 Solution of Differential/Integral Equations\_\_\_\_\_\_\_\_\_\_\_3%**  Newton’s divided difference, forward-difference and backward-difference formulae, Hermite interpolation, numerical differentiation, integration and their error estimates, rectangular rule, trapezoidal rule, Simpson’s one-three and three-eight rules | **9%** |
| **13.** | **MECHANICS:**  **13.1** Particle kinematics, radial and transverse components of velocity and acceleration, circular motion, motion with a uniform acceleration, the Newton laws of motion (the inertial law, the force law and the reaction law), the newtonian model of gravitation, simple-harmonic motion, damped oscillations, conservative and dissipative systems, driven oscillations, nonlinear oscillations **\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_3%**  **13.2** Hamilton’s principle, lagrangian and hamiltonian dynamics, symmetry and conservation laws, central-force motion, two-body problem, orbit theory, Kepler’s laws of motion (the law of ellipses, the law of equal areas, the harmonic law), satellite motion, kinematics of two-particle collisions, motion in non-inertial reference frame, rigid-body dynamics, motion of a rigid body, **\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_3%** | **6%** |
| **14.** | **MATHEMATICAL STATISTICS AND PROBABILITY:**  **14.1 Mathematical Statistics:**  Measure of central tendency (mean ,median, mode, geometric and harmonic means) Distributions(, binomial, piosson, normal, hyper geometric), estimation of parameters, estimation of mean, variance, confidence intervals, hypothesis testing and decision making, types of errors in tests, quality control, control charts for mean, standard deviation, variance, range, goodness of fit, chi-square test, regression analysis, method of least squares, Probability | **5%** |
|  | **Total** | **100%** |